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Art. XII.—*The Psychrometric Formula.*

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In a paper published last year by Dr. Love and myself[1] we discussed a modification proposed by Ekholm to be made in the formula for the wet-and-dry-bulb hygrometer, which would have important consequences if confirmed. The formula so modified would be

$$x = \eta f - AB\ (t - t'),$$

where x and f are respectively the actual vapour-pressure in the atmosphere and the saturation vapour-pressure at the temperature t' of the wet-bulb. A is the ordinary psychrometric constant, and η the co-efficient, less than unity, whose insertion Ekholm advises in order to allow for diminution of vapour-pressure at the surface of the wet-bulb by a hygroscopic action of the material covering it.

It was shown that, if there were an appreciable hygroscopic action affecting the temperature of the wet-bulb, a perceptible difference would be observed between several thermometers covered with different materials. The results recorded showed, however, that three wet-bulb thermometers, covered respectively with silk, linen and cotton, agreed in their readings to within the limits of observation, which were 0.05°C, the thermometers being divided into tenth-degrees. In all, 63 sets of observations were given, each set comprising a comparison of the three wet-bulbs with a Regnault condensation hygrometer. By the application of least squares to the modified formula, the value of η was found to be 0.9974, which is unity to the order of approximation possible in such experiments. The conclusions arrived at were that the proposed change was not justified by the evidence which Ekholm himself produced, and that a direct test showed the supposed basis of it to be incorrect.

These observations, taken in May and June, were all under conditions of fairly high humidity, above 50 per cent. in every case, and consequently low values of the difference $t - t'$. In order to test the formula under a more extended range of circumstances, the observations were resumed, and continued until December. No change was made in the conditions of exposure, nor in the method of procedure, both of which were fully described in the former paper. The cover-

1 Proc. Roy. Soc. Victoria 24 (n.s.), Pt. II., 1911.

ings of the wet-bulbs had been kept clean by periodical washing during the first series of experiments, but on resuming this was neglected until August 10, when 31 more sets had been obtained. It was then found that they were considerably soiled, and an examination of the results indicated that this had seriously affected the readings. After cleaning, some superfluous portions were cut away, leaving the bulbs somewhat freer, and thenceforward the materials were regularly washed. The results herein detailed comprise only the observations since August 10, numbering 103; some figures obtained from the other 31 sets will be presented, as evidence of the necessity for clean coverings to the wet-bulbs. During the last month of the work observations were mostly restricted to hot, dry days in order to gain experience of low humidities. The range of temperature and humidity has thus been largely extended, the humidities now varying between 18.01 per cent. and 95.11 per cent., and the dry temperatures between 7.35° and 31.35°C, while the values of $t - t'$ extend up to 14.6°C.

Observations.

The wet-bulb readings are given in Table I., with the differences between them, the thermometers being numbered 2 (linen), 3 (cotton) and 4 (silk), as before.

Table I.

No.	2	3	4	2—3	2—4
1	11.55	11.65	11.70	—0.10	—0.15
2	11.05	11.10	11.05	— .05	0
3	11.65	11.65	11.70	0	— .05
4	9.45	9.40	9.40	.05	.05
5	9.45	9.50	9.45	— .05	0
6	9.85	9.85	9.75	0	.10
7	8.45	8.50	8.50	— .05	— .05
8	9.15	9.10	9.10	.05	.05
9	9.15	9.10	9.10	.05	.05
10	10.55	10.60	10.55	— .05	0
11	9.55	9.60	9.50	— .05	.05
12	11.85	11.85	11.85	0	0
13	12.75	12.65	12.70	.10	.05
14	11.45	11.45	11.50	0	— .05
15	12.45	12.40	12.40	.05	.05
16	11.65	11.55	11.65	.10	0
17	11.15	11.00	11.05	.15	.10
18	12.55	12.45	12.50	.10	.05
19	12.65	12.65	12.70	0	— .05
20	10.65	10.60	10.65	.05	0

No.	2	3	4	2–3	2–4
21	10.45	10.45	10.45	0	0
22	11.25	11.15	11.20	.10	.05
23	10.40	10.40	10.45	0	— .05
24	11.40	11.35	11.45	.05	— .05
25	13.90	13.90	14.00	0	— .10
26	12.95	12.95	12.95	0	0
27	10.65	10.70	10.65	— .05	0
28	8.75	8.80	8.80	— .05	— .05
29	7.35	7.40	7.40	— .05	— .05
30	9.35	9.40	9.40	— .05	— .05
31	11.25	11.25	11.25	0	0
32	13.80	13.90	14.00	— .10	— .20
33	13.50	13.60	13.70	— .10	— .20
34	12.45	12.55	12.60	— .10	— .15
35	7.85	7.90	7.90	— .05	— .05
36	9.95	10.00	10.00	— .05	— .05
37	10.70	10.70	10.70	0	0
38	12.35	12.35	12.40	0	— .05
39	13.35	13.30	13.40	.05	— .05
40	14.00	14.00	13.95	0	.05
41	12,70	12.65	12.70	.05	0
42	12.55	12.55	12.60	0	— .05
43	13.15	13.05	13.10	.10	.05
44	11.35	11.25	11.35	.10	0
45	11.85	11.75	11.85	.10	0
46	13.55	13.55	13.60	0	— .05
47	13.40	13.35	13.40	.05	0
48	14.10	14.00	14.10	.10	0
49	16.60	16.60	16.60	0	0
50	16.55	16.50	16.60	.05	— .05
51	13.90	13.80	13.90	.10	0
52	12.25	12.15	12.20	.10	.05
53	14.50	14.40	14.50	.10	0
54	10.55	10.50	10.55	.05	0
55	10.95	10.90	10.95	.05	0
56	9.65	9.60	9.60	.05	.05
57	11.35	11.25	11.35	.10	0
58	14.30	14.20	14.35	.10	— .05
59	14.90	14.80	14.95	.10	— .05
60	13.90	13.80	13.90	.10	0
61	14.00	13.90	14.00	.10	0
62	12.35	12.35	12.40	0	— .05
63	15.00	14.90	15.00	.10	0

No.	2	3	4	2–3	2–4
64	7.75	7.70	7.80	.05	— .05
65	9.75	9.70	9.80	.05	— .05
66	10.85	10.80	10.85	.05	0
67	8.85	8.85	8.90	0	— .05
68	9.65	9.65	9.65	0	0
69	9.45	9.40	9.40	.05	.05
70	9.25	9.20	9.20	.05	.05
71	9.05	9.00	9.10	.05	— .05
72	9.55	9.50	9.60	.05	— .05
73	12.75	12.65	12.70	.10	.05
74	12.60	12.55	12.70	.05	— .10
75	13.60	13.50	13.60	.10	0
76	13.15	13.05	13.15	.10	0
77	13.60	13.55	13.70	.05	— .10
78	13.40	13.40	13.50	0	— .10
79	14.10	14.00	14.10	.10	0
80	10.55	10.60	10.65	— .05	— .10
81	8.95	9.00	9.00	— .05	— .05
82	11.45	11.35	11.45	.10	0
83	11.95	11.90	12.00	.05	— .05
84	13.15	13.05	13.20	.10	— .05
85	11.45	11.35	11.50	.10	— .05
86	17.20	17.10	17.20	.10	0
87	9.15	9.00	9.00	.15	.15
88	15.10	15.00	15.10	.10	0
89	18.00	17.80	17.90	.20	.10
90	16.80	16.70	16.80	.10	0
91	17.50	17.30	17.40	.20	.10
92	13.50	13.40	13.60	.10	— .10
93	12.75	12.60	12.60	.15	.15
94	12.80	12.65	12.70	.15	.10
95	15.10	15.10	15.10	0	0
96	16.10	16.10	16.10	0	0
97	16.50	16.50	16.40	0	.10
98	16.20	16.10	16.15	.10	.05
99	19.25	19.25	19.30	0	— .05
100	15.60	15.50	15.80	.10	— .20
101	12.40	12.40	12.50	0	— .10
102	15.80	15.80	16.00	0	— .20
103	16.10	16.10	16.20	0	— .10

The differences are again very slight, and by no means comparable with those which would correspond to any appreciable hygroscopic action. In the case of linen and cotton, out of 103 pairs, 27 show no difference, and only 6 differ by more than 0.1°; the mean difference is 0.04°, while the observable limit is 0.05°. In the other case of linen and silk, 36 show a zero difference, and only 9 show above 0.1° difference, while the average is 0.015°. Combining these with the observations already published, we find the average differences in the two cases to be respectively 0.03° and 0.01°, while out of the large number of differences only 10 in one case and 11 in the other are greater than a tenth of a degree.

We must conclude from these figures that the value of the coefficient η is not perceptibly dependent on the nature of the covering used for the wet-bulb thermometer, this result holding true throughout a considerable range of temperature above zero, and under practically all conditions of humidity.

The full details of the observations are contained in the next table. Under the heading t' are given the mean values, where necessary, of the wet-bulb readings; the pressure is given corrected for temperature.

TABLE II.

No.	t	t'	Dew-point	B	Wind
1	16.60	11.65	6.80	765.8	Fresh N.
2	17.40	11.05	3.85	764.8	Light N.
3	15.20	11.65	8.20	761.0	Strong N.
4	10.55	9.40	7.80	765.5	Calm.
5	11.80	9.45	6.85	761.5	Light S. E.
6	14.20	9.80	4.40	764.6	Light S. E.
7	11.45	8.50	5.35	766.1	Very light S.E.
8	13.65	9.10	4.15	764.8	Very light S.E.
9	11.60	9.10	5.95	764.8	Calm.
10	14.75	10.55	5.00	760.6	Gentle N.
11	14.75	9.55	5.55	759.5	Light N.
12	16.70	11.85	7.95	762.6	Strong N.
13	18.90	12.70	6.85	761.4	Strong N.
14	17.60	11.45	6.55	761.6	Very strong N.
15	20.35	12.40	5.45	760.1	Strong N.
16	18.60	11.60	5.75	760.7	Very strong N.
17	17.50	11.05	4.75	753.4	N. breeze.
18	19.60	12.50	6.10	750.8	N. breeze.
19	14.60	12.65	10.70	751.7	Calm.
20	12.20	10.65	8.60	762.9	Very light S.
21	12.65	10.45	8.50	761.3	Very light S.
22	12.40	11.20	9.85	763.3	Very light S.
23	11.85	10.40	9.25	763.2	Very light S.
24	12.50	11.40	10.65	762.6	Calm.
25	17.30	13.95	11.15	758.8	N. breeze

No.		t		t'		Dew-point		B		Wind
26	-	17.90	-	12.95	-	9.15	-	756.3	-	Light N.
27	-	12.70	-	10.65	-	8.30	-	758.3	-	Calm.
28	-	11.40	-	8.80	-	6.55	-	758.1	-	N. breeze.
29	-	11.60	-	7.40	-	2.25	-	765.4	-	Calm.
30	-	14.55	-	9.40	-	3.65	-	761.8	-	Very light N.
31	-	15.85	-	11.25	-	7.35	-	759.2	-	Light N.
32	-	18.40	-	13.90	-	9.60	-	757.5	-	Calm.
33	-	20.45	-	13.60	-	7.35	-	751.8	-	Fresh N.
34	-	20.75	-	12.55	-	4.35	-	750.0	-	Light N.
35	-	10.85	-	7.90	-	5.55	-	756.4	-	S. breeze.
36	-	12.20	-	10.00	-	7.80	-	765.3	-	Light S.
37		13.35	-	10.70	-	6.15	-	764.3	-	Calm.
38	-	16.15	-	12.35	-	9.70	-	760.7	-	Very light N.
39	-	18.30	-	13.35	-	8.65	-	758.5	-	Very light N.
40	-	17.90	-	14.00	-	10.50	-	758.5	-	Calm.
41	-	18.20	-	12.70	-	7.95	-	756.8	-	Gentle S.
42	-	16.05	-	12.55	-	9.80	-	758.7	-	Calm.
43	-	17.55	-	13.10	-	10.10	-	757.5	-	Calm.
44	-	13.45	-	11.30	-	9.50	-	760.2	-	Very light S.
45	-	16.15	-	11.80	-	8.15	-	758.8	-	Very light S.
46	-	20.25	-	13.55	-	7.45	-	760.3	-	Fresh N.
47	-	19.55	-	13.40	-	7.45	-	759.1	-	Very light N.
48	-	20.05	-	14.05	-	8.40	-	757.6	-	Gentle N.
49	-	21.05	-	16.60	-	13.55	-	754.0	-	Light N.
50	-	21.80	-	16.55	-	12.95	-	751.4	-	Gentle N.
51	-	17.00	-	13.85	-	11.35	-	754.2	-	Gentle S.
52	-	15.25	-	12.20	-	9.60	-	759.8	-	Gentle S.
53	-	20.95	-	14.45	-	9.80	-	756.6	-	N. breeze.
54	-	14.55	-	10.55	-	6.25	-	760.3	-	Light N.
55	-	14.25	-	10.95	-	8.45	-	764.4	-	Light S.W.
56	-	13.45	-	9.60	-	5.95	-	764.7	-	Gentle S.
57	-	15.25	-	11.30	-	7.85	-	762.9	-	Very light S.
58	-	18.60	-	14.30	-	11.35	-	762.1	-	Very light S.
59	-	21.20	-	14.90	-	9.40	-	761.2	-	Very light N.
60	-	16.60	-	13.85	-	12.05	-	760.0	-	Very light S.
61	-	16.80	-	13.95	-	12.15	-	759.4	-	Very light S.
62	-	15.35	-	12.35	-	9.75	-	756.0	-	Light N.
63	-	19.25	-	14.95	-	11.05	-	753.5	-	Gentle, variable.
64	-	12.70	-	7.75	-	2.45	-	762.7	-	Fresh S.W.
65	-	14.75	-	9.75	-	3.75	-	761.0	-	Gentle W.
66	-	15.85	-	10.85	-	5.60	-	759.5	-	Light S.
67	-	13.15	-	8.85	-	4.40	-	763.2	-	Light S.
68	-	14.25	-	9.65	-	3.15	-	762.3	-	Light, variable.

No		t		t'		Dew point		B		Wind
69	-	13.15	-	9.40	-	5.85	-	764.8	-	S. breeze.
70	-	14.20	-	9.20	-	3.65	-	765.0	-	S. breeze.
71	-	13.85	-	9.05	-	4.05	-	766.2	-	Light S.
72	-	14.35	-	9.55	-	4.85	-	765.5	-	Light S.
73	-	19.00	-	12.70	-	6.20	-	761.9	-	Strong N.
74	-	19.80	-	12.60	-	5.45	-	761.0	-	Strong N.
75	-	22.45	-	13.55	-	5.95	-	758.6	-	Strong N.
76	-	21.95	-	13.10	-	5.05	-	758.1	-	Light N.
77	-	23.60	-	13.60	-	5.25	-	756.4	-	Strong N.
78	-	25.65	-	13.45	-	3.85	-	754.8	-	Strong N.
79	-	24.40	-	14.05	-	4.95	-	754.8	-	S.W. breeze.
80	-	15.15	-	10.60	-	6.70	-	755.0	-	Light S.
81	-	14.05	-	9.00	-	4.15	-	759.8	-	Light S.W.
82	-	15.55	-	11.40	-	7.40	-	765.9	-	Very light S.
83	-	15.75	-	11.95	-	8.90	-	765.0	-	Gentle S
84	-	17.20	-	13.15	-	8.95	-	765.5	-	Light S.
85	-	15.50	-	11.45	-	7.45	-	766.0	-	Gentle S.
86	-	29.90	-	17.15	-	5.75	-	755.2	-	Strong N.
87	-	13.75	-	9.05	-	5.35	-	768.9	-	S. breeze.
88	-	23.80	-	15.05	-	8.60	-	758.7	-	Light N.
89	-	31.20	-	17.90	-	7.35	-	755.3	-	Light N.
90	-	31.35	-	16.75	-	4.15	-	755.3	-	Gentle N.
91	-	30.95	-	17.40	-	5.95	-	755.3	-	Gentle N.
92	-	18.20	-	13.50	-	9.40	-	759.5	-	Gentle S.
93	-	17.85	-	12.65	-	7.90	-	759.0	-	Light S.
94	-	17.10	-	12.70	-	8.80	-	758.6	-	Light S.W.
95	-	24.35	-	15.10	-	7.85	-	754.5	-	Strong N.
96	-	27.40	-	16.10	-	6.95	-	752.1	-	Strong N.
97	-	23.60	-	16.45	-	10.40	-	755.6	-	Light S.W.
98	-	21.90	-	16.15	-	11.55	-	754.7	-	Very light S.W.
99	-	28.10	-	19.25	-	13.90	-	756.6	-	Light N.
100	-	26.10	-	15.65	-	7.05	-	745.2	-	Strong N.
101	-	21.90	-	12.45	-	2.60	-	752.5	-	Very light N.
102	-	24.55	-	15.85	-	9.60	-	749.8	-	Gentle N.
103	-	27.90	-	16.15	-	5.65	-	749.5	-	Very light S.W.

Method of Reduction.

In dealing with the former series of observations, the individual values of the constant A were determined from the formula

$$x = f - \mathrm{AB}(t - t'),$$

taking the vapour-pressures x and f from Broch's tables. The arithmetic mean of these values was taken to be the most probable, or most suitable, value of A to be used in the formula. The same method was followed at first in reducing the present series, and considerable time was spent in grouping and examining the values of A before it was discovered that the method was incorrect in principle, though yielding approximately correct results so long as the number of observations was not too small.

By the application of the method of least squares to the more general formula

$$x = \eta f - AB(t - t'),$$

in which there are two constants, η and A, to be determined, a single value of each is obtained to represent the whole set of observations. The corresponding formula with these numerical co-efficients has the property that it gives the value of x with the least probable error from observations of the values of the other quantities concerned. Now this is evidently the result which is required; in practice the psychrometer is used alone, and we are required to determine from its readings the actual value x of the vapour-pressure in the air. We therefore seek a formula of the recognised type with such numerical co-efficients that the value of x will in the long run of similar trials be given with the smallest possible margin of error. If the simple formula

$$x = f - AB(t - t'),$$

be used, or, in other words, if the co-efficient η be assumed to be unity, this end will be attained by a direct application of least squares to the equation as it stands, and not to the severally determined values of the constant A. Taking the arithmetic mean of the values of A is an application of least squares which makes the errors of A a minimum, instead of those of x. The correct value of A which is appropriate to the whole set of observations is therefore given by the equation

$$A \Sigma z^2 = \Sigma (f - x) z,$$

where z is put for convenience in place of B $(t - t')$. Since the individual value of A is given for each observation by

$$A = (f - x)/z,$$

the correct result is the same as would be obtained by weighting the individual values proportionally to z^2, that is practically to $(t - t')^2$. Since $t - t'$ is frequently small, and is in the denominator, this makes it seem probable that the correct value for the present purpose would also be a better value than the simple mean, if the object were to determine A with the least margin of error. (This latter might be the case, for example, if the formula were assumed to be absolute and not merely an approximation of varying accuracy: then the value of A might be considered as an aid to investigation of the properties of air or water-vapour. It need hardly be said that such a procedure would be absurd.)

When this method of reduction was first recognised as more correct, it was not known that any other experimenter had used it. But on investigation, the same method, with a small alteration, the reason for which I could not discover, was found given without remark in Ferrel's paper on "Psychrometrical Tables,"[1] though there are some obvious misprints. The method used by Regnault and most others is not stated in their papers: on the other hand, Angot[2], Pernter[3] and Svensson[4] certainly used arithmetic mean values. For this reason I have thought it well to call attention to the discrepancy between the two methods, though the actual results may not be much different in a good series of observations. For investigating the effects of wind-velocity and other circumstances, the same method is appropriate, and was used in all further study of my own observations.

Results.

The value of A derived from the 103 observations recorded in this paper is 0.0007232 ± 0.0000048. Taking the mean of individual values, the result is higher—viz., 0.0007330; but the probable errors of a single observation of x in the two cases are respectively 0.229 mm. and 0.231 mm., so that the difference in the value of A is of small practical moment. The value given in the previous paper was 0.0007228; using the more correct method of reduction it would become 0.0007167. From the whole set of 166 observations taken together the resulting value is 0.0007227 ± 0.0000043. These various values all agree when only two significant figures are taken, and that is all that can be regarded as really valuable. The final result is then that A = 0.00072, with a probable error of about half a unit in the last place, i.e., $(72 \pm \frac{1}{2}) \times 10^{-5}$. The equation thus becomes

$$x = f - 0.00072\mathrm{B}(t - t').$$

Applying the two-constant formula, the values of η and of A are found to be 0.9877 and 0.0006967 respectively. For the 63 observations of the earlier series, η had the value 0.9974. The lower value now obtained might be regarded as due to incomplete saturation of the air leaving the wet-bulb, or some similar failure in the action which is assumed in theory to occur. But it seemed scarcely likely that this would be more noticeable in the present series than in the other one, since the later observations were distinctively superior in other respects. In order to determine whether the lessened value might be due to the observations at low humidity, the series was

1. Ferrel, Report of Secretary of War, U.S., 1886, vol. iv., p. 233
2. Angot, J. de Physique, 1, 1882, p. 119
3. Pernter, Sitzungsber. Wiener Akad. 87, 1883
4. Svensson, Meteor. Zeitschr. 1896, p. 201.

divided into two groups, in which the humidities were all above and all below 50 per cent. respectively. In the earlier series all humidities were above 50 per cent. The application of the formula to each group separately gave $\eta = 0.9589$ below 50 per cent. humidity and 0.9715 above. Both these values are less than that obtained for the two groups combined, instead of being one greater and one less, as we might expect. Such a result seems to indicate very clearly that the value of the second constant η is almost entirely dependent on the nature and distribution of the accidental errors of the first constant A, so that it will vary arbitrarily with the particular group of observations chosen. In other words, there is no physical justification for the insertion of a second constant. It is probable that its value, determined by trial as above, would always be less than unity, but this does not indicate the existence of any phenomenon which is not implicitly allowed for by the simple formula. The following tables will show that the insertion of it is not attended by any increase in accuracy, provided the observations are good. It may possibly be permissible to say that a series of observations which yields a value of η markedly different from unity is unsatisfactory in some respect.

In Table III. the values of x observed are compared with those deduced from the two formulae, according to the equations

$$x_1 = f - 0.0007232\mathrm{B}(t - t')$$
$$\text{and} \quad x_2 = 0.9877f - 0.0006967\mathrm{B}(t - t'),$$

and the differences, or errors, $\Delta x_1 = x - x_1$ and $\Delta x_2 = x - x_2$, are also given.

TABLE III.

No.	x	x_1	x_2	Δx_1	Δx_2
1	7.36	7.46	7.43	—0.10	—0.07
2	6.01	6.29	6.30	— .28	— .29
3	8.10	8.25	8.19	— .15	— .09
4	7.88	8.14	8.06	— .26	— .18
5	7.39	7.52	7.45	— .13	— .06
6	6.24	6.59	6.57	— .35	— .33
7	6.67	6.64	6.60	.03	.07
8	6.13	6.09	6.08	.04	.05
9	6.95	7.23	7.17	— .28	— .22
10	6.51	7.17	7.13	— .66	— .62
11	6.76	6.01	6.01	.75	.75
12	7.96	7.66	7.62	.30	.34
13	7.39	7.51	7.50	— .12	— .11
14	7.24	6.67	6.68	.57	.56
15	6.71	6.34	6.37	.37	.34
16	6.86	6.31	6.33	.55	.53

No.	x	x_1	x_2	Δx_1	Δx_2
17	6.40	6.29	6.29	.11	.11
18	7.02	6.92	6.94	.10	.08
19	9.58	9.83	9.74	— .25	— .16
20	8.32	8.68	8.60	— .36	— .28
21	8.27	8.21	8.13	.06	.14
22	9.05	9.24	9.14	— .19	— .09
23	8.69	8.59	8.50	.10	.19
24	9.54	9.42	9.33	.12	.21
25	9.87	10.00	9.92	— .13	— .05
26	8.63	8.39	8.35	.24	.28
27	8.16	8.42	8.34	— .26	— .18
28	7.24	7.00	6.96	.24	.28
29	5.37	5.35	5.34	.02	.03
30	5.92	5.94	5.94	— .02	— .02
31	7.65	7.40	7.38	.25	.27
32	8.90	9.34	9.29	— .44	— .39
33	7.65	7.86	7.85	— .21	— .20
34	6.22	6.37	6.41	— .15	— .19
35	6.76	6.33	6.29	.43	.47
36	7.88	7.92	7.86	— .04	.02
37	7.05	8.12	8.05	—1.07	—1.00
38	8.96	8.59	8.54	0.37	0.42
39	8.35	8.67	8.63	— .32	— .28
40	9.45	9.74	9.67	— .29	— .22
41	7.96	7.91	7.89	.05	.07
42	9.02	8.90	8.84	.12	.18
43	9.20	8.77	8.72	.43	48
44	8.84	8.79	8.71	.05	.13
45	8.07	7.91	7.87	.16	.20
46	7.70	7.86	7.85	— .16	— .15
47	7.70	8.05	8.04	— .35	— .34
48	8.21	8.63	8.60	— .42	— .39
49	11.54	11.61	11.53	— .07	.01
50	11.10	11.15	11.08	— .05	.02
51	10.00	10.05	9.97	— .05	.03
52	8.90	8.89	8.83	.01	.07
53	9.02	8.67	8.65	.35	.37
54	7.10	7.28	7.24	— .18	— .14
55	8.24	7.92	7.86	.32	.38
56	6.95	6.77	6.74	.18	.21
57	7.91	7.79	7.75	.12	.16
58	10.00	9.75	9.69	.25	.31
59	8.78	9.12	9.10	— .34	— .32

No.	x	x_1	x_2	Δx_1	Δx_2
60	10.47	10.26	10.17	.21	.30
61	10.54	10.27	10.18	.27	.36
62	8.99	9.04	8.97	— .05	.02
63	9.80	10.29	10.21	— .49	— .41
64	5.44	5.13	5.13	.31	.31
65	5.97	6.24	6.23	— .27	— .26
66	6.78	6.92	6.90	— .14	— .12
67	6.24	6.09	6.07	.15	.17
68	5.72	6.39	6.38	— .67	— .66
69	6.90	6.71	6.67	.19	.23
70	5.92	5.89	5.89	.03	.03
71	6.09	5.92	5.91	.17	.18
72	6.44	6.21	6.20	.23	.24
73	7.07	7.45	7.45	— .38	— .38
74	6.71	6.89	6.90	— .18	— .19
75	6.95	6.66	6.70	.29	.25
76	6.53	6.36	6.40	.17	.13
77	6.62	6.11	6.17	.51	.45
78	6.01	4.81	4.91	1.20	1.10
79	6.49	6.27	6.33	0.22	0.16
80	7.31	7.03	7.00	.28	.31
81	6.13	5.78	5.77	.35	.36
82	7.67	7.73	7.70	— .06	— .03
83	8.49	8.30	8.24	.19	.25
84	8.52	9.01	8.96	— .49	— .44
85	7.70	7.82	7.78	— .12	— .08
86	6.86	7.58	7.65	— .72	— .79
87	6.67	5.97	5.95	.70	.72
88	8.32	7.91	7.92	.41	.40
89	7.65	7.98	8.05	— .33	— .40
90	6.13	6.20	6.32	— .07	— .19
91	6.95	7.36	7.45	— .41	— .50
92	8.78	8.93	8.88	— .15	— .10
93	7.94	8.04	8.01	— .10	— .07
94	8.43	8.51	8.46	— .08	— .03
95	7.91	7.71	7.74	.20	.17
96	7.44	7.45	7.51	— .01	— .07
97	9.39	10.00	9.98	— .61	— .59
98	10.13	10.50	10.45	— .37	— .32
99	11.81	11.73	11.70	.08	.11
100	7.50	7.58	7.63	— .08	— .13
101	5.50	5.61	5.67	— .11	— .17
102	8.90	8.66	8.68	.24	.22
103	6.81	7.27	7.33	— .46	— .52

The errors in the two formulae are never far different ; summing them without regard to sign the results are 27.12 and 27.05, for the 103 observations, so that the average errors are indistinguishable from one another. The same is true of the probable errors deduced by least squares, being 0.229 mm. in each case.

The corresponding values of humidity with the errors of each are given in the following table : —

TABLE IV.

No.	r	r_1	r_2	Δr_1	Δr_2
1	52.42	53.13	52.92	—0.71	—0.50
2	40.72	42.61	42.68	—1.89	—1.96
3	63.08	64.25	63.79	—1.17	—0.71
4	83.12	85.86	85.02	—2.74	—1.90
5	71.75	73.01	72.33	—1.26	—0.58
6	51.83	54.73	54.57	—2.90	—2.74
7	66.30	66.00	65.61	0.30	0.69
8	52.75	52.41	52.32	0.34	0.43
9	68.41	71.16	70.57	—2.75	—2.16
10	52.20	57.50	57.18	—5.30	—4.98
11	54.21	48.20	48.20	6.01	6.01
12	56.33	54.21	53.93	2.12	2.40
13	45.56	46.30	46.24	—0.74	—0.68
14	48.43	44 62	44.68	3.81	3.75
15	37.80	35.72	35.89	2.08	1.91
16	43.09	39.64	39.76	3.45	3.33
17	43.07	42.33	42.33	0.74	0.74
18	41.44	40.85	40.97	0.59	0.47
19	77.57	79.60	78.87	—2.03	—1.30
20	78.71	82.12	81.36	—3.41	—2.65
21	75.94	75.39	74.66	0.55	1.28
22	84.50	86.27	85.34	—1.77	—0.84
23	84.12	83.16	82.28	0.96	1.84
24	88.50	87.38	86.55	1.12	1.95
25	67.28	68.17	67.62	—0.89	—0.34
26	56.63	55.05	54.79	1.58	1.84
27	74.73	77.11	76.37	—2.38	—1.64
28	72.18	69.79	69.39	2.39	2.79
29	52.85	52.66	52.56	0.19	0.29
30	48.09	48.25	48.25	—0.16	—0.16
31	57.17	55.31	55.16	1.86	2.01
32	56.62	59.41	59.10	—2.79	—2.48
33	42.83	44.01	43.95	—1.18	—1.12
34	34.19	35.02	35.24	—0.83	—1.05

No.	r	r_1	r_2	Δr_1	Δr_2
35	69.91	65.46	65.05	4.45	4.86
36	74.55	74.93	74.36	—0.38	0.19
37	61.90	71.29	70.68	—9.39	—8.78
38	65.69	62.98	62.61	2.71	3.08
39	53.46	55.51	55.25	—2.05	—1.79
40	62.01	63.91	63.45	—1.90	—1.44
41	51.29	50.97	50.84	0.32	0.45
42	66.57	65.68	65.24	0.89	1.33
43	61.74	58.86	58.52	2.88	3.22
44	77.07	76.63	75.94	0.44	1.13
45	59.16	57.99	57.70	1.17	1.46
46	43.65	44.56	44.50	—0.91	—0.85
47	45.59	47.66	47.60	—2.07	—2.01
48	47.13	49.54	49.37	—2.41	—2.24
49	62.28	62.66	62.22	—0.38	0.06
50	57.25	57.50	57.14	—0.25	0.11
51	69.44	69.79	69.24	—0.35	0.20
52	69.10	69.02	68.56	0.08	0.54
53	49.00	47.09	46.99	1.91	2.01
54	57.68	59.14	58.81	—1.46	—1.13
55	68.21	65.56	65.07	2.65	3.14
56	60.59	59.02	58.76	1.57	1.83
57	61.41	60.48	60.17	0.93	1.24
58	62.81	61.24	60.87	1.57	1.94
59	46.98	48.80	48.69	—1.82	—1.71
60	74.57	73.08	72.44	1.49	2.13
61	74.17	72.27	71.64	1.90	2.53
62	69.37	69.75	69.21	—0.38	0.16
63	59.14	62.10	61.62	—2.96	—2.48
64	49.82	46.98	46.98	2.84	2.84
65	47.87	50.04	49.96	—2.17	—2.09
66	50.67	51.72	51.57	—1.05	—0.90
67	55.47	54.13	53.96	1.34	1.51
68	47.35	52.90	52.81	—5.55	—5.46
69	61.33	59.64	59.29	1.69	2.04
70	49.17	48.92	48.92	0.25	0.25
71	51.74	50.30	50.21	1.44	1.53
72	53.00	51.11	51.03	1.89	1.97
73	43.32	45.65	45.65	—2.33	—2.33
74	39.13	40.17	40.23	—1.04	—1.10
75	34.46	33.02	33.22	1.44	1.24
76	33.37	32.50	32.70	0.87	0.67
77	30.61	28.25	28.53	2.36	2.08

No	r	r_1	r_2	Δr_1	Δr_2
78	24.59	19.68	20.09	4.91	4.50
79	28.60	27.63	27.90	0.97	0.70
80	57.11	54.92	54.69	2.19	2.42
81	51.43	48.49	48.41	2.94	3.02
82	58.42	58.87	58.64	—0.45	—0.22
83	63.84	62.41	61.96	1.43	1.88
84	58.44	61.80	61.45	—3.36	—3.01
85	58.82	59.74	59.43	—0.92	—0.61
86	21.90	24.19	24.42	—2.29	—2.52
87	57.06	51.07	50.90	5.99	6.16
88	38.01	36.14	36.18	1.87	1.83
89	22.67	23.64	23.85	—0.97	—1.18
90	18.01	18.21	18.57	—0.20	—0.56
91	20.88	22.12	22.39	—1.24	—1.51
92	56.57	57.54	57.22	—0.97	—0.65
93	52.27	52.93	52.73	—0.66	—0.46
94	58.18	58.73	58.39	—0.55	—0.21
95	34.97	34.08	34.22	0.89	0.75
96	27.45	27.49	27.71	—0.04	—0.26
97	43.41	46.23	46.14	—2.82	—2.73
98	51.92	53.82	53.56	—1.90	—1.64
99	41.83	41.55	41.45	0.28	0.38
100	29.87	30.19	30.39	—0.32	—0.52
101	28.19	28.75	29.06	—0.56	—0.87
102	38.88	37.83	37.92	1.05	0.96
103	24.40	26.05	26.26	—1.65	—1.86

These figures again show no decided gain in accuracy by the use of the modified formula. The sums of the errors taken without regard to sign are 182.34 and 180.98, showing an average error 0.01 less in one case than in the other. The probable errors are respectively 1.57 and 1.54, the difference being too small to be significant.

Wind-effects.

The value 0.00072 for the constant A was derived from observations taken under all conditions of wind-velocity, no artificial method of ventilation having been used. By grouping the observations in accordance with the strength and direction of the wind, however, the value of the constant is found to vary in the manner generally acknowledged. It is a maximum under calm conditions and decreases as the velocity of the wind rises. From only 11 observations in calm air A = 0.0007887, which agrees with the value 0.0007882 from 17 observations of the earlier series (the values given in the former paper were obtained by taking the arithmetic means). With strong winds, say from 15 to 20 miles an hour (about 25 ft. a second) upwards, the result of 25 observations is that A = 0.0006936. Taking together all those observations in which the air was distinctly in motion, the value is 0.0007213. Thus it would seem that the value 0.00072 is satisfactory for all conditions of ventilation, supposed occurring arbitrarily, while absolute calm should be avoided if possible.

The suggestion previously made that a difference might be found according to the direction of the wind is not confirmed by these observations. In 47 cases the wind was from the north (including N.E. and N.W.), and for these A = 0.0007186; in 42 cases the wind was from the south, and the value comes 0.0007199.

Any relations which may exist between the value of A and the temperature or humidity were completely masked by the wind-effect, and the number of observations was not sufficient to allow of a separation.

Effect of Soiled Coverings.

That the nature of the evaporating surface does not affect the temperature attained by the wet-bulb thermometer is a conclusion which does not extend to the state of cleanliness of that surface. The necessity for frequent renewal or cleansing of the materials is well recognised, and the observations which, as already mentioned, were unintentionally made with soiled coverings, show that this regulation is by no means unimportant. The different wet-bulbs agreed together as closely as when clean, all being equally soiled, or at any rate exposed to the same conditions, but the observations were discordant among themselves, i.e., in the individual values of A, several were erroneous to an extreme, and the wet-bulbs read in practically all cases too high. The value of A derived from the 31 observations was 0.0008684, with a probable error of 0.0000232, while with the two-constant formula the result was $\eta = 0.9254 \pm 0.0155$, $A = 0.0005892 \pm 0.0000631$. The hindrance to evaporation was evidently large, and the instrument in such a state is useless.

In closing, I should like to offer my thanks to both Dr. Love and Professor Lyle for the kindly advice and assistance which have always been placed at my disposal.